Derivation of Kinetic Energy Equation
for
2-Wing Revolving Door with Showcase & Curved Panel

Warren F. Davis, Ph.D.
Davis Associates, Inc.
43 Holden Road
Newton, MA 02465-1909

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Herein is derived the equation for the rotational kinetic energy of a 2-wing revolving door equipped with a showcase and concentric curved panel attached to the outer end of each wing. The following assumptions are made that characterize the door.

1. The rotating elements of the door correspond to those illustrated in the plan view shown in Figure 1 and are assumed to rotate together as a rigid unit.

2. The mass of each flat panel is distributed uniformly in the horizontal direction across the width of the panel. This assumption is not necessary for the curved panels because all points within each curved panel lie at the same distance R from the axis of rotation.

3. It is not necessary, and it is not assumed, that the panel masses are distributed uniformly in the direction parallel to the axis of rotation. That is, in the vertical direction.

4. The thickness of all panels is assumed to be a small fraction of the horizontal dimension of the panel.

5. Other components of the door that may be in motion, such as the ceiling, showcase floor and the drive mechanism, are not considered.

Method – The total rotational kinetic energy of the door is the sum of the rotational kinetic energies of the individual constituent panels, the energy of each of which is given by the basic formula

$$E = \frac{1}{2} I \omega^2 , \tag{1}$$

where I is the moment of inertia of the respective panel about the fixed axis of rotation located at the geometric center of the door, and ω is the angular velocity of rotation of the door in radians/second.

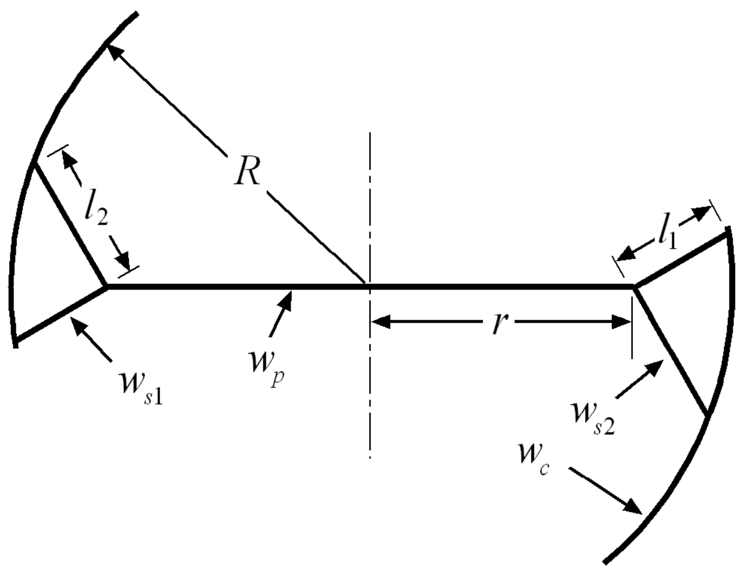

Figure 1. Plan view of the revolving elements of the 2-wing revolving door.

As a consequence of the assumptions delineated on page 1, the moment of inertia of the radial panels, of width *r* and identified by the subscript *p* in Figure 1, is given by the well-known formula for the moment of inertia of a uniform bar of mass m_p and length *r* rotating about one end

$$I_p = \frac{1}{3} m_p r^2 = \frac{1}{3} \frac{w_p}{g} r^2 , \tag{2}$$

where w_p is the weight of the radial panel and g is the acceleration of gravity at sea level, which has the approximate value of 32.15 ft/sec^2. Newton's first law relating force, mass and acceleration has been used in (2) to express the mass of the panel m_p in terms of the weight of the panel w_p at sea level.

Since the elements of mass of the curved panels identified by the subscript c in Figure 1 all lie at the same distance R from the axis of rotation, the moment of inertia of each curved panel is simply

$$I_c = m_c R^2 = \frac{w_c}{g} R^2 , \tag{3}$$

where again Newton's law has been used to express the mass of the curved panel in terms of its weight at sea level.

The moments of inertia about the axis of rotation at the geometric center of the door of the remaining flat showcase panels of widths l_1 and l_2 and identified by the subscripts $s1$ and $s2$ in Figure 1, are derived using the well-known fact that the total moment of inertia of an arbitrary mass distribution about an arbitrary axis of (rigid) rotation is equal to (a) the moment of the distribution about the center of mass *plus* (b) the moment about the axis of rotation with the total mass of the distribution concentrated at the center of mass.

The moment (a) about the center of mass of each showcase panel is given by the well-known formula for a bar of uniform mass

$$I = \frac{1}{12} m l^2 , \tag{4}$$

where m is the mass of the bar and l the length of the bar. In this application, m is the mass of the respective showcase panel and l is its width.

To determine the moments (b), it is necessary first to have an expression for the distance of the center of mass of each showcase panel from the axis of rotation at the center of the door. Because the masses of the showcase panels are assumed to be uniform in the horizontal direction, the center of mass of each such panel is located midway along the panel. One end of each panel lies at a distance r from the axis of rotation, and the other end at a distance R. If the width of the panel is l, it is necessary, therefore, to develop an expression for the distance ρ illustrated in Figure 2.

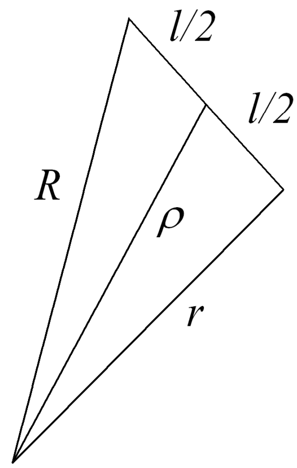

Figure 2. Distance ρ to center of mass of showcase panel.

Treating R, r, ρ and l as vectors, the perimeter triangle in Figure 2 satisfies the relationship

$$r^2 = R^2 + l^2 + 2R \cdot l$$

where "·" represents the vector inner, or "dot", product and the square of a vector represents the square of the magnitude of the vector. Likewise, the upper triangle in Figure 2 satisfies the relationship

$$\rho^2 = R^2 + \frac{l^2}{4} + R \cdot l\,.$$

When the first of these relationships is used to eliminate $R \cdot l$ from the second, the result is

$$\rho^2 = \frac{1}{2}\left(r^2 + R^2\right) - \frac{l^2}{4}\,. \tag{5}$$

Using (4) and (5), the moment of inertia of the first showcase panel with respect to the axis of rotation at the center of the door is

$$I_1 = \frac{1}{12} m_{s1} l_1^2 + m_{s1}\left[\frac{1}{2}\left(r^2 + R^2\right) - \frac{l_1^2}{4}\right],$$

or

$$I_1 = \frac{1}{2}\frac{w_{s1}}{g}\left(r^2 + R^2\right) - \frac{1}{6}\frac{w_{s1}}{g} l_1^2 . \qquad (6)$$

where Newton's law has again been used to express the mass of the panel in terms of its weight at sea level.

In exactly the same way, the total moment of inertia of the second showcase panel about the axis of rotation at the geometric center of the door is

$$I_2 = \frac{1}{2}\frac{w_{s2}}{g}\left(r^2 + R^2\right) - \frac{1}{6}\frac{w_{s2}}{g} l_2^2 . \qquad (7)$$

The total moment of inertia of all rotating components of the door illustrated in Figure 1, taking into account also that there are two identical sections to the door, is twice the sum of (2), (3), (6) and (7). That is

$$I = \frac{2}{g}\left[w_c R^2 + \frac{w_{s1} + w_{s2}}{2}\left(r^2 + R^2\right) - \frac{1}{6}\left(w_{s1} l_1^2 + w_{s2} l_2^2\right) + \frac{1}{3} w_p r^2\right]. \qquad (8)$$

The expression (1) for the rotational kinetic energy of the door can be expressed in terms of the rotation rate of the door in revolutions per minute (rpm) by substituting in (1) the relationship

$$\omega = \Omega\left(\frac{2\pi}{60}\right)$$

where Ω is the rotation rate of the door in revolutions per minute. That is,

$$E = \frac{1}{2} I \left(\frac{2\pi}{60}\right)^2 \Omega^2 . \qquad (9)$$

When (8) is substituted for I in (9) and the numerical values of π and g utilized, the final result for the rotational kinetic energy of the 2-wing revolving door is

$$E = \frac{1}{2931.7}\left[w_c R^2 + \frac{w_{s1} + w_{s2}}{2}\left(r^2 + R^2\right) - \frac{1}{6}\left(w_{s1} l_1^2 + w_{s2} l_2^2\right) + \frac{1}{3} w_p r^2 \right]\Omega^2 \quad (10)$$

where the rotational kinetic energy E is in units of pound-feet; the weights w_c, w_{s1}, w_{s2} and w_p of the door components are in pounds; the dimensions r, R, l_1 and l_2 are in feet; and the rotation rate of the door Ω is in revolutions per minute.[1]

Equation (10) can be solved for the rotation rate of the door in revolutions per minute Ω at which the door exhibits 2.5 lb-ft and 7.0 lb-ft of rotational kinetic energy E. The results are

$$\Omega_{2.5} = \frac{85.6}{\sqrt{w_c R^2 + \frac{w_{s1} + w_{s2}}{2}\left(r^2 + R^2\right) - \frac{1}{6}\left(w_{s1} l_1^2 + w_{s2} l_2^2\right) + \frac{1}{3} w_p r^2}} \quad (11)$$

and,

$$\Omega_{7.0} = \frac{143}{\sqrt{w_c R^2 + \frac{w_{s1} + w_{s2}}{2}\left(r^2 + R^2\right) - \frac{1}{6}\left(w_{s1} l_1^2 + w_{s2} l_2^2\right) + \frac{1}{3} w_p r^2}} \quad (12)$$

[1] Note that the approximate value of $g = 32.15$ ft/sec^2 has been used in this derivation. The frequently used poorer approximation $g = 32$ ft/sec^2 will alter the result slightly. Namely, the constant in the denominator of the lead fraction in (10) becomes 2918.0. The result (10) is the more accurate and is to be preferred.