Kinetic Energy Equations for *n*-wing Revolving Doors with a Cylindrical Core

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Herein are derived the equations for the rotational kinetic energy of *n*-wing revolving doors with a cylindrical core. The following assumptions are made that characterize the door.

1. The rotating part of the door consists of a number *n* of identical flat rectangular radially directed panels each rigidly attached along one vertical edge to the outer surface of a hollow vertical cylindrical core.

2. The core to which the panels are attached revolves around a vertical axis coincident with the geometric center of the cylindrical core.

3. The mass of each panel is distributed uniformly in the horizontal direction across the width of the radial panels and the circumference of the cylindrical core. It is not necessary, and it is not assumed, that the panel masses, radial and core, are distributed uniformly in the direction parallel to the axis of rotation. That is, in the vertical direction.

4. The thickness of the radial panels is a small fraction of their width, and the thickness of the cylindrical panel defining the core is a small fraction of the circumference of the core.

5. Other components of the door that may be in motion, such as a ceiling and the drive mechanism, are not considered.

The figure below illustrates the relevant dimensions used in the derivation.

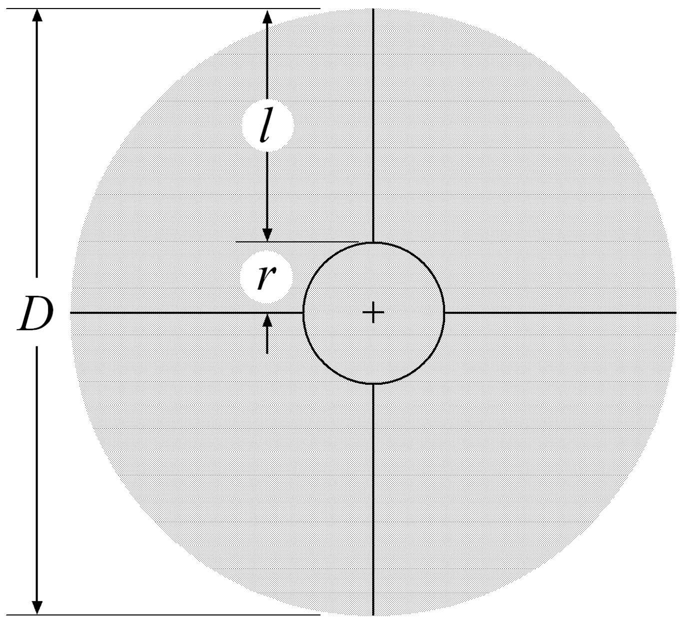

Figure 1. Multiple wing revolving door with a cylindrical core.

D represents the overall diameter of the door, *r* the radius of the cylindrical core, and *l* the width of each radial panel. The total number of radial panels is arbitrary and is given by *n*.

The rotational kinetic energy of any mass distribution associated with rigid rotation of the mass distribution about a fixed axis is

$$E = \frac{1}{2} I \omega^2 , \tag{1}$$

where *I* is the moment of inertia of the mass distribution about the fixed axis of rotation, and ω is the angular velocity of the rotation in radians/second.[1]

As a consequence of the assumptions delineated on page 1, the moment of inertia of each radial door panel about its center-of-mass is equivalent to that of a uniform bar rotating about its center. That is, to

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[1] Specifically, the moment of inertia is the second spatial moment of the mass distribution about the axis of rotation.

$$I = \frac{1}{12} M L^2 \tag{2}$$

where M is the mass of the bar and L is its length. Because of the equivalency just mentioned, M is here the mass of an individual radial door panel and L its width. Combining (1) and (2), the rotational kinetic energy of a radial panel of mass M and width L due to its rotation about its own center-of-mass is

$$E^{(cm)} = \frac{1}{24} M L^2 \omega^2 \,. \tag{3}$$

The kinetic energy due to the rotation of the center-of-mass of the panel about the axis of rotation of the door is

$$E^{(orbit)} = \frac{1}{2} M \rho^2 \omega^2 \tag{4}$$

where ρ is the radial distance of the center-of-mass of the panel from the axis of rotation of the door.

The total kinetic energy of each radial panel due to its rigid rotation within the structure of the door is the sum of the two components given by (3) and (4).[2] Namely,

$$E^{(panel)} = E^{(cm)} + E^{(orbit)} \,. \tag{5}$$

From Figure 1, it is obvious that

$$\frac{D}{2} = l + r \qquad \text{or} \qquad r = \frac{D}{2} - l \tag{6}$$

Since the distribution of mass across the width of the radial panels is uniform, the center-of-mass of the radial panels is located at the center (in the horizontal direction) of the panel. That is, at a distance $l/2$ from the attachment point of the radial panel to the

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[2] "Rigid" here means that, due to the fact that the panel is rigidly attached to the structure of the door, it is forced to rotate about its own center-of-mass as well as to rotate as a whole about the axis of rotation of the door. Equation (5) accounts for both of these rotational components that characterize the motion of the panel.

cylindrical core. Given that the radius of the cylindrical core is r, the radial distance of the center-of-mass of a radial panel from the axis of rotation at the center of the core is

$$\rho = r + \frac{l}{2} = \left(\frac{D}{2} - l\right) + \frac{l}{2} = \frac{D-l}{2}, \tag{7}$$

where (6) has been introduced for r.

Let the mass of the cylindrical core be m_c and that of a radial panel m_p. From (4) and (7), the kinetic energy associated with the rotation of the center-of-mass of a radial panel about the axis of rotation of the door is

$$E^{(orbit)} = \frac{1}{2} m_p \omega^2 \rho^2 = \frac{1}{8} m_p \omega^2 \left(D-l\right)^2 . \tag{8}$$

And, from (3), the kinetic energy associated with the rotation of a radial panel about its own center-of-mass is

$$E^{(cm)} = \frac{1}{24} m_p \omega^2 l^2 . \tag{9}$$

Therefore, from (5), (8) and (9), and the fact that there are n radial panels in all, the total kinetic energy associated with the revolving radial panels is

$$E^{(panels)} = \frac{n}{24} m_p \omega^2 \left[3\left(D-l\right)^2 + l^2\right] = \frac{n}{24} m_p \omega^2 \left(3D^2 - 6Dl + 4l^2\right). \tag{10}$$

Since the cylindrical core is assumed to be hollow, the entire mass m_c of the core is concentrated at the radius r. Therefore, the kinetic energy associated with the rotation of the core is

$$\begin{aligned} E^{(core)} &= \frac{1}{2} m_c \omega^2 r^2 = \frac{1}{2} m_c \omega^2 \left(\frac{D}{2} - l\right)^2 = \\ &= \frac{1}{8} m_c \omega^2 \left(D-2l\right)^2 = \frac{1}{8} m_c \omega^2 \left(D^2 - 4Dl + 4l^2\right), \end{aligned} \tag{11}$$

where (6) has been used for r.

The total rotational kinetic energy of the door is the sum of (10) and (11). Namely,

$$E = E^{(panels)} + E^{(core)} = \frac{\omega^2}{24}\left[n m_p\left(3D^2 - 6Dl + 4l^2\right) + 3m_c\left(D^2 - 4Dl + 4l^2\right)\right]. \qquad (12)$$

Grouping by descending powers of D yields

$$E = \frac{\omega^2}{24}\left[D^2\left(3n m_p + 3m_c\right) - 6Dl\left(n m_p + 2m_c\right) + 4l^2\left(n m_p + 3m_c\right)\right]. \qquad (13)$$

And taking out a factor of 12 gives

$$E = \frac{1}{2}\omega^2\left[\left(\frac{D}{2}\right)^2\left(n m_p + m_c\right) - Dl\left(\frac{n}{2}m_p + m_c\right) + l^2\left(\frac{n}{3}m_p + m_c\right)\right]. \qquad (14)$$

To express this kinetic energy in units of pound-feet, the units used by the ANSI A156.27 national standard for automatic revolving doors, the masses m_p and m_c must be expressed in "equivalent" pounds and the width of the radial panel l and the door diameter D expressed in feet. Also, the angular velocity ω must be expressed as equivalent revolutions per second for compatibility with ANSI A156.27.

The pound is *not* a unit of mass, but a unit of force. But, with the understanding that weight in pounds is determined at sea level, the weight w in pounds that corresponds to the mass m (in slugs) of an individual door panel at sea level is given by Newton's first law, which relates force and mass, as

$$w = mg\,, \quad \text{or} \quad m = \frac{w}{g}\,, \qquad (15)$$

where g is the acceleration of gravity at sea level and has the approximate value 32.15 ft/sec^2.

Since one revolution is equal to 2π radians, and a minute contains 60 seconds,

$$\omega = \left(\frac{2\pi}{60}\right)\Omega = \left(\frac{\pi}{30}\right)\Omega\,, \qquad (16)$$

where Ω is the rotation rate of the door in revolutions per minute (rpm).

Substituting (15) and (16) into (14) and using the numerical values for g and π gives,

$$E = \frac{1}{5863}\left[\left(\frac{D}{2}\right)^2\left(n\,w_p + w_c\right) - D\,l\left(\frac{n}{2}w_p + w_c\right) + l^2\left(\frac{n}{3}w_p + w_c\right)\right], \tag{17}$$

where the total rotational kinetic energy E of the door is in units of pound-feet (lb-ft), w_p and w_c are the weights of a radial panel and the total weight of the core, respectively, in pounds, l is the width of a radial panel in feet, D the overall diameter of the door in feet, n is the number of radial panels and Ω is the rotation rate of the door in revolutions per minute (rpm).[3]

Equation (17) can be used to calculate the rotation rate Ω of the door that results in the door carrying exactly 2.5 lb-ft of rotational kinetic energy by solving (17) for Ω with E set to 2.5 lb-ft. The result is,

$$\Omega_{2.5} = \frac{121}{\sqrt{\left(\frac{D}{2}\right)^2\left(n\,w_p + w_c\right) - D\,l\left(\frac{n}{2}w_p + w_c\right) + l^2\left(\frac{n}{3}w_p + w_c\right)}}\,. \tag{18}$$

Equation (17) may likewise be used to determine the rotation rate Ω of the door that results in the door carrying exactly 7.0 lb-ft of rotational kinetic energy by solving (17) for Ω with E set to 7.0 lb-ft. The result is,

$$\Omega_{7.0} = \frac{203}{\sqrt{\left(\frac{D}{2}\right)^2\left(n\,w_p + w_c\right) - D\,l\left(\frac{n}{2}w_p + w_c\right) + l^2\left(\frac{n}{3}w_p + w_c\right)}}\,. \tag{19}$$

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[3] The authors of ANSI A156.27-2003 indicate that the kinetic energy of the door is expressed in foot-pounds (ft-lb). However, a foot-pound is a unit of torque. Though dimensionally equivalent, the correct unit of kinetic energy is the pound-foot (lb-ft).