RADIATION — *Waves and Quanta*[1]

Note of **Louis de Broglie**, presented by Jean Perrin.

(Translated from *Comptes rendus*, Vol. 177, 1923, pp. 507-510)

Let us consider a material moving object of rest mass m_0 moving with respect to a fixed observer with a speed $\upsilon = \beta c \quad (\beta < 1)$. According to the principle of the inertia of energy, it should possess an internal energy equal to $m_0 c^2$. On the other hand, the quantum principle suggests associating this internal energy with a simple periodic phenomenon of frequency ν_0 such that

$$h\nu_0 = m_0 c^2,$$

c being, as usual, the limiting velocity of the theory of relativity and h Planck's constant.

For the fixed observer, the frequency $\nu = \dfrac{m_0 c^2}{h\sqrt{1-\beta^2}}$ corresponds to the total energy of the moving object. But, if this fixed observer observes the internal periodic phenomenon of the moving object, he will see it slowed down and will attribute to it a frequency $\nu_1 = \nu_0\sqrt{1-\beta^2}$; for him this phenomenon varies therefore like

$$\sin 2\pi\nu_1 t \, .$$

Now let us suppose that at the time $t = 0$ the moving object coincides in space with a wave of frequency ν defined above and propagating in the same direction as it does with the speed $\dfrac{c}{\beta}$. This wave, which has a speed greater than c, cannot correspond to

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[1] Concerning the present note, see Brillouin, *Comptes rendus*, Vol. 168, 1919, p. 1318.

transport of energy; we will only consider it as a fictitious wave associated with the motion of the object.

I maintain that, if at the time $t = 0$, there is phase agreement between the vectors of the wave and the internal phenomenon of the object, this phase agreement will be maintained. In effect, at time t the object is at a distance from the origin equal to $\upsilon t = x$; its internal motion is then represented by $\sin 2\pi \nu_1 \frac{x}{\upsilon}$.

The wave, at this point, is represented by

$$\sin 2\pi \nu \left(t - \frac{x\beta}{c} \right) = \sin 2\pi \nu x \left(\frac{1}{\upsilon} - \frac{\beta}{c} \right).$$

The two sines are equal and the phase agreement is realized if one has

$$\nu_1 = \nu (1 - \beta^2),$$

a condition that is clearly satisfied by the definitions of ν and ν_1.

The demonstration of this important result rests uniquely on the principle of special relativity and on the correctness of the quantum relationship as much for the fixed observer as for the moving observer.

Let us apply this to an atom of light. I showed elsewhere[2] that the atom of light should be considered as a moving object of a very small mass ($< 10^{-50}$ g) that moves with a speed very nearly equal to c (although slightly less). We come therefore to the following conclusion: *The atom of light, which is equivalent by reason of its total energy to a radiation of frequency ν, is the seat of an internal periodic phenomenon that, seen by the fixed observer, has at each point of space the same phase as a wave of frequency ν propagating in the same direction with a speed very nearly equal (although very slightly greater) to the constant called the speed of light.*

[2] See *Journal de Physique*, 6-th series, Vol. 3, 1922, p. 422.

Let us consider now the case of an electron describing a closed trajectory with uniform speed slightly less than c. At time $t = 0$, the object is at point O. The associated fictitious wave, launched from the point O and describing the entire trajectory with the speed $\frac{c}{\beta}$, catches up with the electron at time τ at a point O' such that $\overline{OO'} = \beta c \tau$.

One has then that

$$\tau = \frac{\beta}{c}[\beta c(\tau + \mathrm{T}_r)] \quad \text{or} \quad \tau = \frac{\beta^2}{1-\beta^2}\mathrm{T}_r\ ,$$

where T_r is the period of revolution of the electron in its orbit. The internal phase of the electron, when the electron goes from O to O', has a variation of

$$2\pi\nu_1\tau = 2\pi\frac{m_0 c^2}{h}\mathrm{T}_r\frac{\beta^2}{\sqrt{1-\beta^2}}\ .$$

It is *almost necessary* to suppose that the trajectory of the electron will be stable *only if* the fictitious wave passing O' catches up with the electron in phase with it: the wave of frequency ν and speed $\frac{c}{\beta}$ has to be in resonance over the length of the trajectory. This leads to the condition

$$\frac{m_0\beta^2 c^2}{\sqrt{1-\beta^2}}\mathrm{T}_r = nh\ , \qquad\qquad n \text{ being integer.}$$

Let us show that this stability condition happens to be that of the Bohr and Sommerfeld theories for a trajectory described by a constant speed. Let us call p_x, p_y, p_z the momenta of the electron along three rectangular axes. The general condition for stability formulated by Einstein is in effect

$$\int_0^{T_r} (p_x\,dx + p_y\,dy + p_z\,dz) = nh \qquad (n \text{ integer})^{3}$$

which, in the present case, can be written

$$\int_0^{T_r} \frac{m_0}{\sqrt{1-\beta^2}} (\upsilon_x^2 + \upsilon_y^2 + \upsilon_z^2)\,dt = \frac{m_0 \beta^2 c^2}{\sqrt{1-\beta^2}} T_r = nh \, ,$$

as above.

In the case of an electron turning in a circular orbit of radius R with an angular velocity ω, one finds again for sufficiently small speeds the original formula of Bohr: $m_0 \omega R^2 = n \frac{h}{2\pi}$.

If the speed varies along the length of the trajectory, one finds again the Bohr-Einstein formula if β is small. If β assumes large values, the question becomes more complicated and necessitates a special examination.

Pursuing research along these lines we have reached important results, which will be communicated soon. We are as of today able to explain the phenomena of diffraction and of interference taking into account the quantization of light.

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[3] The case of quasi-periodic motion does not present any new difficulty. The necessity of satisfying the condition stated in the text for an infinity of pseudo-periods leads to the conditions of Sommerfeld.

Translated by Brigitte and Barton Lane and transcribed by Warren F. Davis, January, 1978.